

Abstract

The McKay-Miller-Širáň (MMS) graphs H_q , defined for each prime power q greater than 2, are a family of large graphs with diameter 2 and degree $(3q - \delta)/2$, where $q = 4k + \delta$, which were introduced by McKay, Miller, and Širáň in 1998. Notably, the Hoffman-Singleton graph is isomorphic to the McKay-Miller-Širáň graph H_5 .

MMS graphs were initially constructed as lifts of $K_{q,q}$ using voltages from the additive group of $GF(q)$. Later, Šiagiová provided their alternative construction as lifts of two-vertex graphs for $q = 4k + 1$. Moreover, Hafner described a geometric construction of MMS graphs via biaffine planes, which he used to determine the automorphism groups of all MMS graphs.

Using Hafner's results, we identify a specific abelian automorphism group of order q^2 in MMS graph H_q . We show that this group acts semiregularly on the vertex set of H_q , partitioning it into two orbits. This observation leads to a proof that every MMS graph can be realized as a lift of a two-vertex graph, i.e., it is a bi-Cayley graph.

Motivated by a problem proposed by Rosa, Šiagiová and Meszka found a set of five edge-disjoint Hoffman-Singleton graphs in K_{50} sharing a common automorphism group of order 25. We characterize the voltage assignments that generate the Hoffman-Singleton graph from Šiagiová's construction and we find a correspondence between these voltage assignments and ovals in the classical projective plane of order 5.

By utilizing this characterization and employing computational methods, we establish that there are 16 non-isomorphic sets of five edge-disjoint Hoffman-Singleton graphs that share an automorphism group of order 25, which stem from the unique quintuple of mutually disjoint ovals in the classical projective plane of order 5, which have the line at infinity as a common tangent line.

Finally, we briefly consider the related concept of q disjoint ovals sharing a common tangent in the classical projective plane of order q .

Keywords: McKay-Miller-Širáň graphs, Hoffman-Singleton graph, graph lifts, bi-Cayley graphs, ovals in projective planes