Abstract

We analyze the equations of motion and the constitutive relations in the wavenumber domain, and we present a heterogeneous finite-difference (FD) formulation of the equations for four configurations of a wavefield in an elastic isotropic medium. The spatial discretization implies that the medium must be Nyquist-wavenumber band-limited. We investigate consequences of the spatial discretization and the Nyquist-wavenumber band limitation of the entire equations. The wavenumber band limitation of the equations replaces spatial derivatives by continuous spatial convolutions. The latter means that the wavenumber band limitation removes discontinuities of the spatial derivatives of particle velocity and stress at the material interface. The continuous spatial convolution must be approximated by a discrete convolution sum of a finite extent, that is by a proper FD operator. Additionally, we investigate the exact and simulated wavefields in the wavenumber domain. We show that the wavefield in an arbitrary heterogeneous model has an unbounded wavenumber spectrum. The wavenumbers higher than the Nyquist wavenumber cannot be represented in the FD grid, which causes an error of the discrete representation of the wavefield. Due to multiplication of spatially dependent material parameters with wavefield variables, the error propagates from the high wavenumbers to the entire wavenumber spectrum. We explain why this error, as well as the error due to aliasing, are small enough. In addition to that, we also address physical consequences of a theoretical assumption that the true medium is wavenumber band limited. We investigate scattering of a wave on a small perturbation, and we provide a physical insight into its mechanism. The analysis yields that for a k_m-wavenumber band-limited medium, the scattered wavefield is accurate only to wavenumber $k_m/2$. This results in a requirement of at least four grid points per shortest wavelength of the wavefield. Finally, we investigate a dependence of an interface error of FD operators on the order of the operator, frequency, a relative position of grid points with respect to the material interface, and on the material interface representation. The numerical analysis shows that a jump of wavespeed at the material interface causes a large interface error in the vicinity of the material interface. The wavenumber band limitation reduces the interface error of the FD operators significantly.

Keywords: material interface, finite-difference method, wavenumber band limitation, wavenumber spectrum, interface error