Abstract

This thesis regards two properties of graphs – cyclic connectivity and flows. They emerged as important invariants of graphs, especially 3-regular ones, and are related to many widely-open problems in graph theory. We divide our results into three areas.

First, we prove that if the order of a (k, g)-cage G is roughly at most twice the Moore bound, then G is cyclically $(k-2)g$-edge-connected and each cycle-separating $(k-2)g$-edge-cut in G separates a g-cycle. In particular, this is true for some parameters k, g, where the (k, g)-cage is unknown, including the potential missing Moore graph of degree 57 and girth 5. This result follows from estimating the size of a k-regular graph with girth at least g and s semiedges. Our results bring a new light in the study of the cage problem.

Second, we consider a cyclically c-edge-connected cubic graph G with a c-edge-cut that separates G into two cyclic components G_1 and G_2. We discuss methods of completing the component G_i to a cyclically c-edge-connected cubic graph. For $c = 5$ and $c = 6$, we prove that such a completion is always possible by adding 3 and 8 new vertices, respectively, unless G_i is the 5-cycle. These results can be applied in inductive proofs or for a computer-assisted generation of cubic graphs with prescribed cyclic connectivity.

Finally, we introduce a new notion of a flow using multidimensional vectors, and propose possible upper and lower bounds on the two-dimensional flow number of bridgeless graphs. To obtain upper bounds, we provide a geometrical representation of two-dimensional flows of cubic graphs.

Keywords: cyclic connectivity, girth, cages, flows, cubic graphs