

Abstract

This thesis investigates Siamese decompositions of complete graphs — partitions of complete graphs into strongly regular subgraphs that share a common subgraph. The most studied class of these decompositions are Siamese color graphs, which are closely related to balanced generalised weighing matrices and pseudo-geometric strongly regular graphs arising from generalised quadrangles. Building on prior work by Kharaghani and Torabi, and by Klin, Reichard, and Woldar, we focus on Siamese color graphs for the case $q = 3$.

We examine the theoretical foundations and the structural properties of the graphs involved in such decompositions. In particular, we study pseudo-geometric strongly regular graphs with a fixed spread S , corresponding distance-regular graphs with diameter 3, and an antipodal system S . We also analyse auxiliary objects used in the construction of Siamese decompositions and identify the cases where no such objects exist.

To support this investigation, we develop computational tools for enumerating and classifying Siamese color graphs, leveraging canonical representations and symmetry considerations. Using our algorithm, we complete the classification of non-isomorphic Siamese color graphs of order 3, thus finishing the process started by Klin, Reichard, and Woldar.

Keywords: Siamese color graph, Siamese decomposition, strongly regular graphs, distance-regular graphs, pseudo-geometric graphs