Kolmogorov Superposition Theorem and its applications in Topological Algebra

Arkady Leiderman

Ben-Gurion University of the Negev, Beer Sheva, ISRAEL

E-mail: arkady@math.bgu.ac.il

Let $[0,1]$ denote the unit closed segment. The classical Kolmogorov Superposition Theorem states that any d-variable continuous function defined on the unit d-dimensional cube $[0,1]^d$ can be represented by means of superpositions of one-variable continuous functions. First, we briefly survey analytical and topological aspects of this celebrated result which resolved the Hilbert's 13th problem.

Our research concerns to the structure of several free objects of Topological Algebra: free topological group $F(X)$, free abelian topological group $A(X)$, free locally convex space $L(X)$, and free topological vector space $V(X)$.

We will show how Kolmogorov Superposition Theorem can be effectively applied for the positive solution of the following question (see [1], [2], [3]):

Question. *Let X be any finite-dimensional metrizable compact space. Is it possible to embed isomorphically $\Phi(X)$ into $\Phi[0,1]$, where Φ is a free topological functor listed above (Φ is A, L, V)?*

We will discuss some relevant problems which are still open. Consider the space $C_p(X)$ of continuous real-valued functions on a compact space X equipped with the topology of pointwise convergence.

Problem. *Find a complete characterization of compact spaces X such that there exists a continuous linear (linear and open) map from $C_p[0,1]$ onto $C_p(X)$.*

The Kolmogorov Superposition Theorem has been considered as a promising tool in Computer Science. We will say a few words about the possible applications of Kolmogorov-Arnold Neural Networks in modern Artificial Intelligence (AI) systems.

[1] A. Leiderman, S.A. Morris and V. Pestov, *The free abelian topological group and the free locally convex space on the unit interval*, J. London Math. Soc. 56 (1997), 529–538.

[2] M. Krupski, A. Leiderman and S.A. Morris, *Embedding of the free abelian topological group $A(X \oplus X)$ into $A(X)$*, Mathematika, 65 (2019), 708–718.

[3] A. Leiderman, S.A. Morris, *Embeddings of free topological vector spaces*, Bull. Austr. Math. Soc. 101 (2020), 311–324.

[4] A. Leiderman, M. Tkachenko, *Embedding the free topological group $F(X^n)$ into $F(X)$*, RACSAM, 118:87 (2024).