Normal edge-colorings of cubic graphs

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Abstract

A k-edge-coloring of a graph G is an assignment of colors $\{1, ..., k\}$ to edges of G, such that adjacent edges receive different colors. If α is an edge-coloring of G, then for a vertex v of G, let $S_\alpha(v)$ be the set of colors that edges incident to v receive. If G is a cubic graph, then a normal k-edge-coloring of G is a k-edge-coloring with the property that for any edge uv $|S_\alpha(u) \cup S_\alpha(v)| \neq 4$. The normal chromatic index of a cubic graph (denoted by $\chi'_N(G)$) is the smallest k, for which G admits a normal k-edge-coloring. Jaeger conjectured in 1985 that all bridgeless cubic graphs have normal chromatic index at most five. Even if the original conjecture of Jaeger is for bridgeless cubic graphs, we can define the normal chromatic index for any simple cubic graph. We show that $\chi'_N(G) \leq 7$ for any simple cubic graph G, and this bound is best-possible. Moreover, we discuss other possible approximation results, where we insist to use exactly five colours but we admit to have some "abnormal" edges.