

IMPROVEMENTS TO MOORE BOUNDS FOR VERTEX-TRANSITIVE GRAPHS OF GIVEN DEGREE AND DIAMETER

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The *Degree/Diameter Problem* is the problem of finding the largest order $n(\Delta, D)$ of a graph of maximum degree Δ and diameter D . The well-known Moore bound, $M(\Delta, D) = 1 + \Delta((\Delta - 1)^D - 1)/(\Delta - 2)$, provides a natural upper bound on $n(\Delta, D)$, and graphs that attain this bound are called Moore graphs. To avoid trivialities we will assume $\Delta \geq 3$ and $D \geq 2$, in which case Moore graphs are very rare. Any graph G of maximum degree Δ and diameter D (a (Δ, D) -graph) is said to have the *defect* $\delta(G) = M(\Delta, D) - |V(G)|$.

We present bounds on the number of cycles of length $2D + 1$ in graph with prescribed degree Δ , diameter D and defect δ which besides the term $\Delta(\Delta - 1)^D/2$ depend only on degree Δ and defect δ .

Using two classical number theory results due to Niven and Erdős, we prove that for any fixed degree $\Delta \geq 3$ and any positive integer δ , the order of a largest vertex-transitive Δ -regular graph of diameter D differs from the Moore bound by more than δ for (asymptotically) almost all diameters $D \geq 2$. We also obtain an estimate for the growth of this difference, or defect, as a function of D .

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